Codimension two bifurcation observed in a phase converter circuit

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Abstract

Bifurcations observed in a phase converter circuit are investigated. We obtain a codimension two bifurcation which is intersection of period-doubling bifurcations. Periodic solutions generated by these bifurcations become chaotic states through a cascade of a codimension three bifurcation which is intersection of D-type of branching and period-doubling bifurcation.

I. Introduction

In this paper we study codimension two bifurcations observed in a forced oscillatory circuit containing saturable inductors, which is called a phase converter circuit, see Fig. 1. The normalized circuit equations are described by

$$\begin{aligned}\frac{dx}{dt} &= y\\ \frac{dy}{dt} &= -ky-(c_0+c_1)x-\tfrac{1}{4}c_3(x^2+3u^2)x\\ \frac{du}{dt} &= v\\ \frac{dv}{dt} &= -kv-(\tfrac{1}{3}c_0+c_1)u-\tfrac{1}{4}c_3(x^2+3u^2)u\\ &\quad +B\cos t+B_0\end{aligned}\tag{1}$$

where

$$x=\phi_a-\phi_b,\ u=\phi_c,\ k=g/\omega C,\ B\propto E,\ B_0\propto E_0$$

In this circuit, the characteristic of inductor is assumed to be a cubic function, i.e., the relation between current i and flux ϕ is assumed to be $i=c_1\phi+c_3\phi^3$. Equations (1) can be considered as a nonlinear coupled system with Duffing's equation and an equation describing a parametric excitation circuit. Hence we may observe various complicated behavior such as chaotic state due to a period-doubling process of parametric excitation, codimension two bifurcations, the coexistence of several periodic oscillations which are correlated with jump and hysteresis behaviors and the fundamental, higher-harmonic and sub-harmonic resonances.

In particular we are interested in codimension two bifurcations. In the neighborhood of a codimension two bifurcation point the dynamical behavior exhibits complicated features and some types of codimension two bifurcations may relate to the generation of chaotic states. Numerical analysis shows that new route to chaos through a cascade of a codimension three bifurcation.

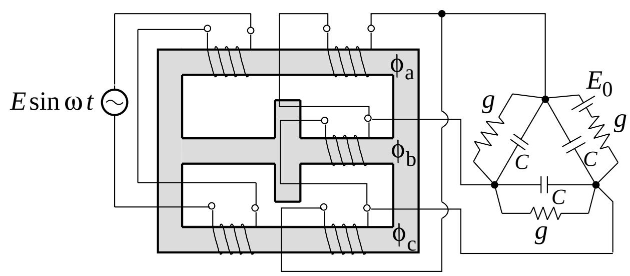

Figure 1: A phase converter circuit.

II. Method of analysis

Equations (1) are rewritten as:

$$\dot{x}=f(t,x,\lambda)\tag{2}$$

In Eqs. (1), f is periodic in t with period 2π:

$$f(t+2\pi,x,\lambda)=f(t,x,\lambda)\tag{3}$$

We assume that Eqs. (1) has a solution $x(t)=\varphi(t,x_0,\lambda)$ with initial condition x_0: $x(0)=\varphi(0,x_0,\lambda)=x_0$. Since f has the period 2π, we can naturally define the Poincaré mapping T_λ from the state space R^4 into itself:

$$T_\lambda: R^4\to R^4; x_0\longmapsto T_\lambda(x_0)=\varphi(2\pi,x_0,\lambda)\tag{4}$$

If a solution $\varphi(t,x_0,\lambda)$ is a periodic, then the point x_0 is a fixed point of T_λ:

$$F_\lambda(x_0) := x_0 - T_\lambda(x_0) = 0 \quad (5)$$

Computation of a periodic solution of Eqs. (1) has now been reduced to find $x_0 \in R^4$ satisfying Eq. (5).

The generic bifurcations of the periodic solution are known as codimension one bifurcations: tangent, period-doubling, Neimark-Sacker bifurcations. At a bifurcation value of parameters, if a periodic solution satisfies two bifurcation conditions, then the bifurcation refers to as a codimension two bifurcation. The degenerate bifurcation: D-type of branching observed in the system which possess a symmetrical property is also called codimension two bifurcation. These bifurcation phenomena can be traced out by solving the fixed point equation and bifurcation condition simultaneously [1].

We use the notation ${}_k\mathrm{D}^m$ (resp. ${}_k\mathrm{I}^m$) denoting a type having even (resp. odd) number of characteristic multipliers on the real axis $(-\infty, -1)$, k indicates the number of characteristic multiplier outside the unit circle in the complex plane, and m indicates m periodic point of T_λ.

III. Results

Now we show some numerical results of bifurcation diagrams and behavior of the solutions observed in Eqs. (1). We use the following notations in bifurcation diagrams: G_k^m, I_k^m and D_k^m represent respectively tangent, period-doubling bifurcation and D-type of branching of m-periodic point and k denotes the number to distinguish several bifurcation sets of same period. In the following we fix several parameters in Eqs. (1) as

(a) $c_0 = c_1 = 0.0$, $c_3 = 1.0$,

(b) $k = 0.1$.

The reasons are as follows:

(a) We choose the same value in [1],

(b) We obtain 3-dimensional bifurcation diagram shown in Fig. 2. When k is lager than 0.3, intersection of period-doubling bifurcations disappear. If k is less than 0.1, bifurcation structure would be more complex.

We consider parameter plane for B and B_0.

In Fig. 3 a stable fixed point with $x = y = 0$ exists in the shaded region. By increasing the parameter B along the line l_1, 2-periodic points with $x = y = 0$ are generated by the period-doubling bifurcation I_1^1, see Fig. 4(a). This bifurcation with $x = y = 0$ is therefore as same as the bifurcation of Duffing's equation shown in [1]. Otherwise when B and B_0 change along the line l_2, the period-doubling bifurcation I_2^1 generates 2-periodic points with $xy \neq 0$, see Fig. 4(b), which corresponds to the parametric excitation phenomenon in the original circuit. At circled points in Fig. 3 the two period-doubling bifurcations I_1^1 and I_2^1 intersect which are called codimension two bifurcation of double period-doubling bifurcations. In the neighborhood of this point D-type of branchings D_2^2 and D_1^2 necessarily appear [2].

Figure 5 shows the detail bifurcation diagram of Fig. 3. In the figure we see successive codimension three bifurcation which is intersection of D-type of branching and period-doubling bifurcation. The mechanism of them are as follows:

(1) Cascade of period-doubling bifurcations $I_1^{2^n}$ $(n = 1, 2, 3)$ of periodic points with $x = y = 0$ occur. Eqs. (1) reduces to Duffing's equation of u and v when $x = y = 0$.

(2) Under the influence of coupling term another period-doubling bifurcation I_2^1 appears. At the intersecting points of I_1^1 and I_2^1, codimension two bifurcation is generated and D-type of branchings: D_1^2 and D_2^2 appear as mentioned above.

(4) $D_1^{2^n}$ and $I_1^{2^n}$ intersect at square marked points which are called codimension three bifurcations. In the neighborhood of these points, D-type of branching of 2^{n+1}- periodic points $D_1^{2^{n+1}}$ and period-doubling bifurcation of 2^n-periodic points $I_2^{2^n}$ necessarily appear, see Fig. 6. The D-type of branching $D_1^{2^{n+1}}$ and the next period-doubling bifurcation $I_1^{2^{n+1}}$ will intersect.

(5) By repeating (4) the successive codimension three bifurcations occur.

In Fig. 7 we see that curves D^{2^n} and I^{2^n} forms a tree branch. This pattern is called tree-like pattern [3] . A similar tree-like pattern is found in a mean value defined on a period-doubling cascade in one-dimensional dynamical system. This property can be explained by a renormalization of a function series on the cascade [4]. In Fig. 7 we observe three types of phase transitions, see Fig. 8. One of them is from a stable periodic point to a chaotic attractor. Others are from periodic points of saddle type. The resulting chaotic states may have both stable and unstable invariant sets. Hence the chaotic state can be called a chaotic saddle and it is unstable.

IV. Concluding Remarks

We have investigated global properties of bifurcation sets of periodic solutions observed in a phase converter circuit. We observed chaotic states from 2-periodic points generated by double period-doubling bifurcations through a cascade of a codimension three bifurcation. The double period-doubling bifurcations are naturally appear in a coupled system which remains bifurcation structures of an uncoupled system. Further research is needed to study a detailed mechanism of generating the double period-doubling bifurcations and classification of the codimension three bifurcation.

References

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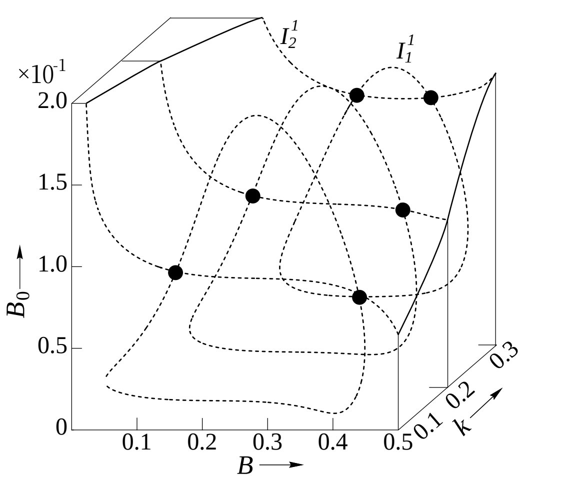


Figure 2: Bifurcation diagram in $B-B_0-k$ space. Dashed curves and circled points indicate period-doubling bifurcations and codimension two bifurcations, respectively.

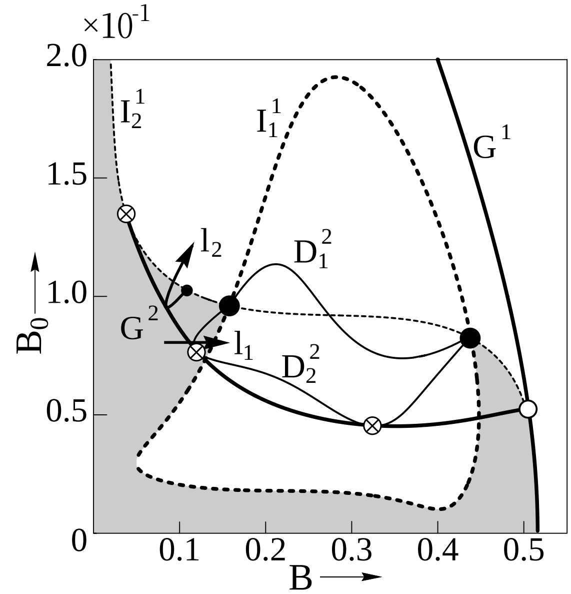
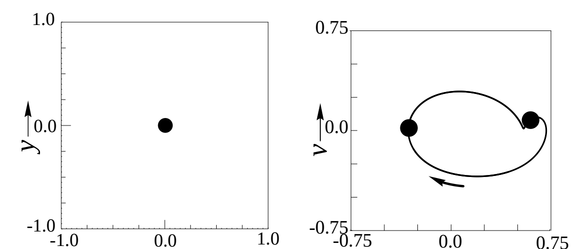
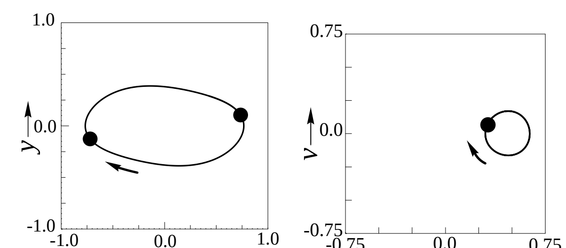


Figure 3: Bifurcation diagram for Eqs. (1). Open circle indicate the codimension two bifurcation called TP-bifurcation in [5]. From this point tangent bifurcation of 2-periodic points appear.



(a) $B = 0.15$, $B_0 = 0.08$.



(b) $B = 0.12$, $B_0 = 0.1$.

Figure 4: Trajectories generated by the period-doubling bifurcations: I_1^1 and I_2^1. The points marked • and the arrow indicate the 2-periodic points of Poincaré map and the direction of trajectory respectively.

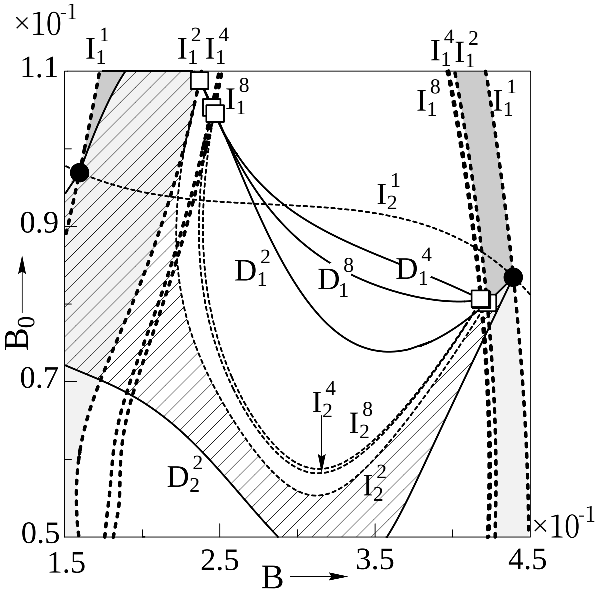


Figure 5: Detailed bifurcation diagram of Fig. 3 . Two cascades of period-doubling bifurcations are observed. The symbols square indicate intersection of D-type of branching and period-doubling bifurcation.

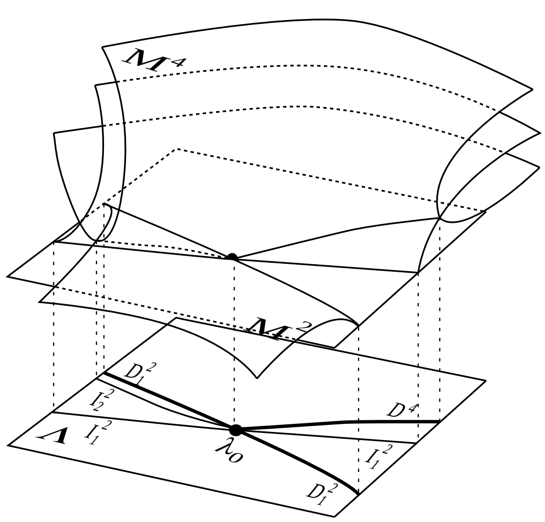


Figure 6: Manifolds of 2-periodic points M^2 and manifolds of 4-periodic points M^4 in the neighborhood of λ_0: intersection of period-doubling bifurcation and D-type of branching. Λ indicates parameter plane.

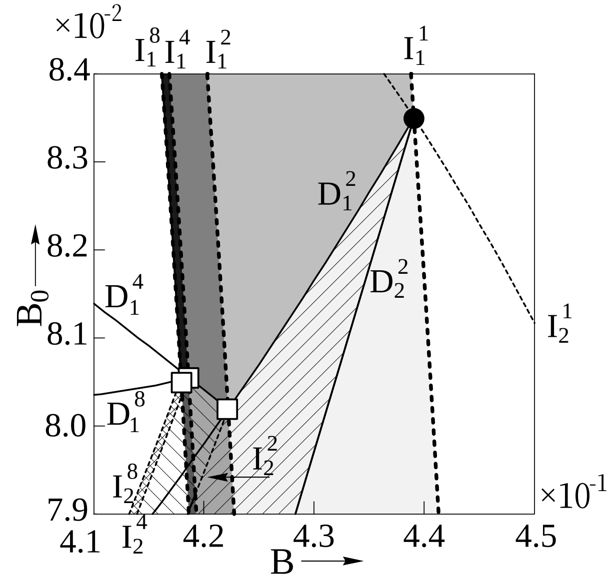


Figure 7: Enlarged bifurcation diagram of Fig.5 .

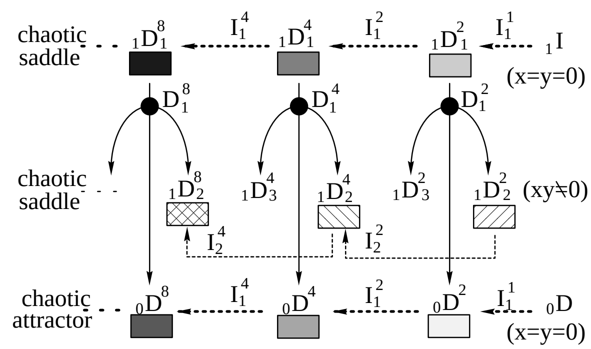


Figure 8: Schematic diagram of phase transitions observed in Fig. 7. Bifurcations written by italic letters and shades are correspond to those of Fig. 7.