SYNCHRONIZED FIRING OF FITZHUGH-NAGUMO NEURONS BY NOISE

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Abstract— *We investigate the influence of noise on the synchronization between the spiking activities of neurons with external impulsive forces. By choosing the appropriate noise intensity and a number of neurons subject to noise we find that small noise can be a promoter of synchronization phenomena in neural activities.*

I. Introduction

A neuron, or the fundamental element of the brain, generates various temporal patterns of spikes. Among such firing patterns, synchronous firing of neurons in connection with neural signal processing has attracted much interest(see [1] and references therein). In particular, the influence of noise on the synchronization [2] are studied for several types of neurons:e.g. Hodgkin-Huxley [3–5] and FitzHugh-Nagumo [6,7], because real neurons can operate accurately even in a noisy environment. However, in these studies, to obtain synchronous firing patterns very large noise intensity is needed.

In this paper we consider globally coupled FitzHugh-Nagumo(FHN) neurons with external noise and periodic impulsive forces. By choosing appropriate noise intensity and a number of neurons subject to noise we find that small noise can be a promoter of synchronization phenomena in neural activities.

II. System Equation

The system equation of electrically (gap junction) coupled FHN neurons is described as

$$\begin{aligned}\frac{dx_i}{dt} &= c\left(x_i - \frac{1}{3}x_i^3 + y_i\right) + h\sum_{k\in Z}\delta(t - 2k\pi/\omega) \\ &\quad - \frac{w}{N-1}\sum_{j=1,j\neq i}^{N}(x_i - x_j) + D_i\xi_i(t) \\ \frac{dy_i}{dt} &= -\frac{1}{c}(x_i + by_i + a) \qquad (1)\end{aligned}$$

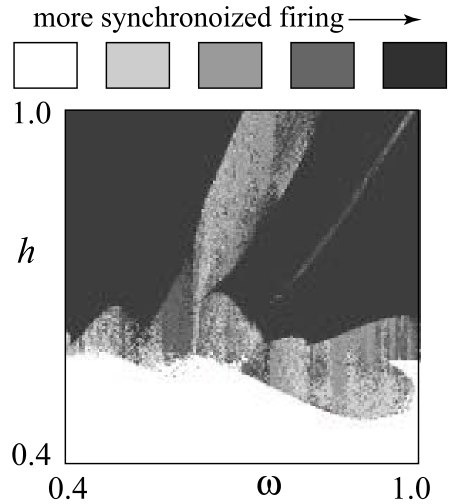


Fig. 1. The parameter region in which synchronous firing is observed. $D_i = 0$.

where i indicates neuron number, $\delta(t)$ is the Dirac's delta function, h and ω are the amplitude and the angular frequency of the impulsive force, respectively, w is the coupling coefficient, $\xi_i(t)$ is the Gaussian white noise with $<\xi_i(t)\ \xi_i(t')>$ $=\delta(t-t')$ and D_i denotes the noise intensity. We use the different noise for each neuron. To generate a random number we use Mersenne Twister method [8]. The values of system parameters are fixed as

$$a = 0.7,\ b = 0.8,\ c = 3.0, w = -0.3 \qquad (2)$$

for the occurrence of a stable equilibrium point in the system of Eq. (1) with $h = 0$ and $D_i = 0$. Equation (1) is numerically integrated by using stochastic Euler method [9,10] with the time step $\Delta t = 2\pi/1024$.

III. Results

In Fig. 1 we show the results of counting a number of neurons with synchronous firing in the parameter space (ω, h). In the darker region we observe more synchronous firing. In the white region, coupled neurons never fire, because it is not

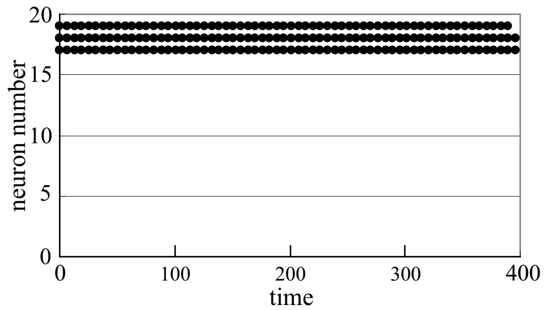


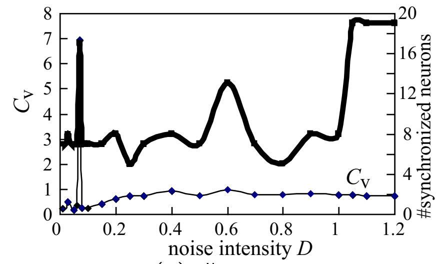
Fig. 2. Time series after the transient time (1024×1000 steps) in Eq. (1) with $D = 0$, $\omega = 0.436$ and $h = 0.592$.

enough impulsive stimulus. In order to obtain more synchronous firing, we inject the noise $\xi_i(t)$ to some FHN neurons in the coupled system. A number of neurons subject to the noise is abbreviated to "#NN". We choose three parameter sets of ω and h from Fig. 1 for the occurrence of few synchronous firing and after adding the noise we study the effect of the noise on the synchronization.

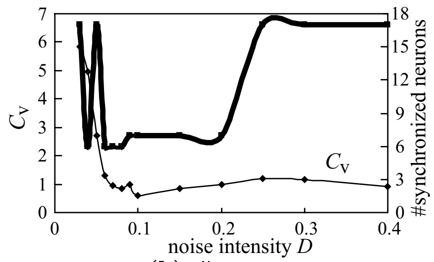
A. Case(1)

We set the values of parameters (ω, h) as (0.436, 0.592). In this parameter setting the raster records of all the firing events in the coupled system without the noise is shown in Fig. 2. From this figure we can see that only three neurons (neuron 17 to 19) generate spikes. After adding the noise we calculate coefficient of variation (C_v) of interspike interval (ISI) for neuron 1 and count a number of neurons with synchronized firing by changing the noise intensity and #NN, see Fig. 3. Thick and thin solid curves indicate the number of neurons with synchronized firing and C_v, respectively.

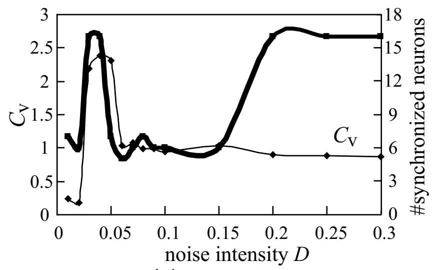
In Fig. 3(a) adding the small intensity noise (D=0.07) to only one neuron (neuron 20), complete synchronized firing of 17 neurons is observed. Figure 4(a) shows firing events as time series data at D=0.07 and #NN=1. From Fig. 3 we can see that when the noise intensity is less than 0.1, the number of neurons with synchronized firing and C_v are increased simultaneously. On the other hand for large intensity of the noise all neurons except noise-injected neurons produce synchronous firing and C_v converges to about 1 which shows irregular spikes [11, 12]. In Fig. 4(b) we show the temporal firing pattern at D=0.4 and #NN=3. At this noise level all neurons



(a) #NN=1.



(b) #NN=3.



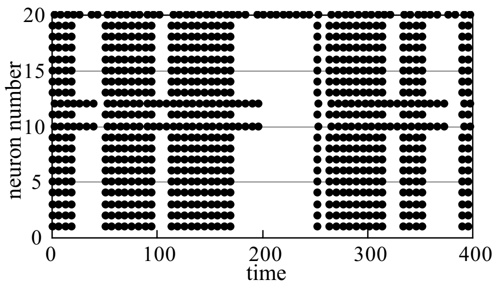
(c) #NN=4.

Fig. 3. Coefficient of variation (C_v) and the number of neurons with synchronous firing as a function of D_i observed in Eq. (1) with $\omega = 0.436$ and $h = 0.592$. After a transient time we calculate them using 40,000 time steps data.

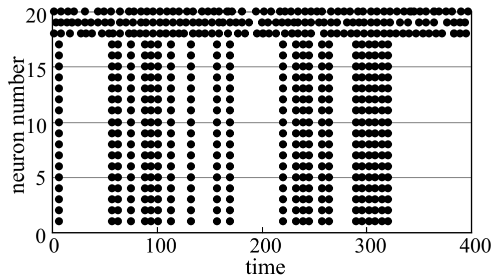
except noise-injected neurons are synchronized. This synchronous firing pattern is robust against adding the small intensity noise (D_i=0.001) to these synchronized 17 neurons, see Fig. 4(c).

B. Case(2)

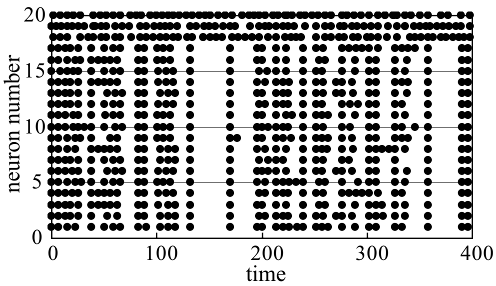
In the second case we set the values of parameters (ω, h) as (0.4, 0.585). Traveling wave is observed when D=0 shown in Fig. 5(a). The order of firing is controllable by changing initial conditions of x_i. Figure 6 shows C_v and the number of neurons with synchronous firing. For small value of the noise intensity C_v is almost zero which means regular spikes. In this case also the increase in C_v and the number of neurons with synchronous firing occur at similar values of the noise intensity. An example of complete synchronized neurons due to an appropriate amount of the noise is shown in Fig. 5(b).



(a) $D_i = 0.07$ and #NN=1.



(b) $D_i = 0.4$ and #NN=3.



(c) D_i=0.001(i=1,$\cdots$,17) and D_i=0.4 (i=18,19,20).

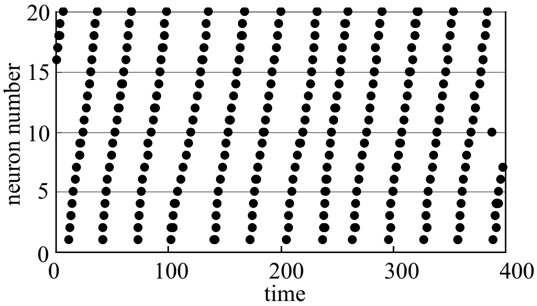
Fig. 4. Temporal firing pattern observed in Eq. (1) with $\omega = 0.436$ and $h = 0.592$ and different noise intensity.

C. Case(3)

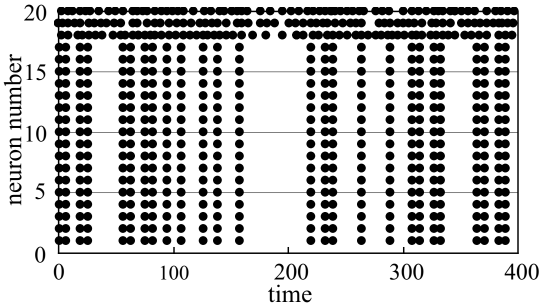
In the third case we set the values of parameters (ω, h) as (0.94, 0.594). Even in the case of the absence of the noise neurons produce irregular spikes, see Fig. 7(a). Figure 8(a) shows that we cannot obtain complete synchronization only by adding the noise to one neuron. By increasing the number of neurons subject to the noise and the noise intensity, complete synchronous firing can be achieved as shown in Fig. 8(b). Although it cannot be achieved, complete synchronous firing is observed in some time interval, see Fig. 7(b).

IV. Conclusions

We have investigated the influence of noise on synchronous firing in globally coupled FHN neurons with external impulsive force. We decided the three values of the amplitude and the angu-

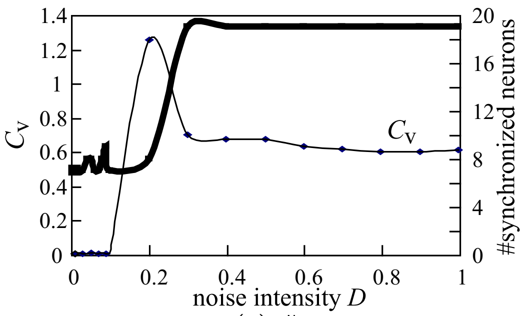


(a) $D = 0.0$.

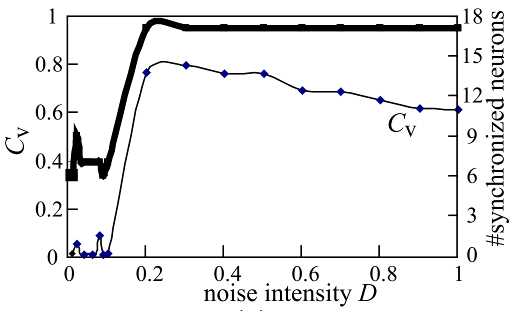


(b) $D = 0.4$ and #NN=3.

Fig. 5. Temporal firing pattern observed in Eq. (1) with $\omega = 0.4$ and $h = 0.585$.

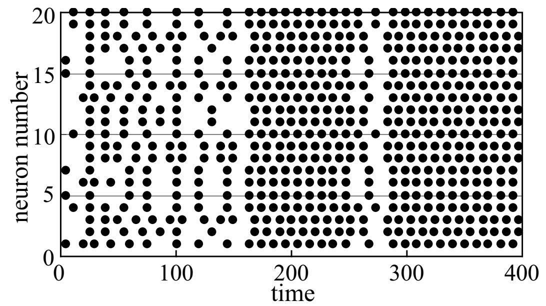


(a) #NN=1.

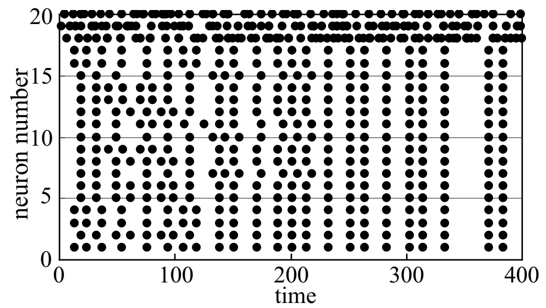


(b) #NN=3.

Fig. 6. C_v and the number of neurons with synchronous firing as a function of D_i observed in Eq. (1) with $\omega = 0.4$ and $h = 0.585$.

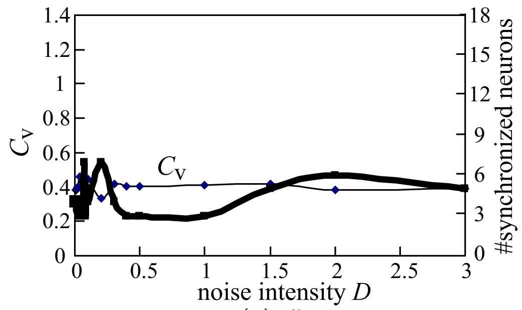


(a) $D = 0.0$.

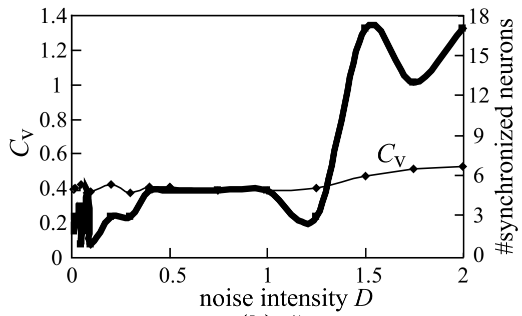


(b) $D = 1.0$ and #NN=3.

Fig. 7. Temporal firing pattern observed in Eq. (1) with $\omega = 0.94$ and $h = 0.594$.



(a) #NN=1.



(b) #NN=3.

Fig. 8. C_v and the number of neurons with synchronous firing as a function of D_i observed in Eq. (1) with $\omega = 0.94$ and $h = 0.594$.

lar frequency of the impulsive force. In each case an appropriate amount of noise (intensity and the number of neurons subject to the noise) can synchronize firing of neurons. This is extension of noise-induced synchronization [9, 13, 14], because the neurons without the noise are indirectly influenced by the noise through the electrical coupling. We calculated C_v and the number of neurons with synchronous firing as a function of the noise intensity and found that for the small noise intensity they are increased simultaneously. It is an open problem to study a system of synaptically coupled neurons containing a time delay.

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