

Synchronization in Synaptically Coupled Neurons with Hub Structure

Ryosuke Ochi¹, Hiroyuki Kitajima² and Eri Ioka³

†Faculty of Engineering, Kagawa University
 2217-20 Hayashi Takamatsu, Kagawa 761-0396, Japan
 Phone: +81-87-864-2226, FAX: +81-87-864-2262

¹Email: s07g452@stmail.eng.kagawa-u.ac.jp

²Email: kitaji@eng.kagawa-u.ac.jp, ³Email: s07d451@stmail.eng.kagawa-u.ac.jp

Abstract—We investigate synchronization in a system of coupled Morris-Lecar (ML) neurons with hub structure. The ML model can be changed class I and class II excitabilities by changing the value of one parameter. Homogeneous (coupled the same class neurons) case is considered for both excitatory and inhibitory synapses. We found that for excitatory or inhibitory coupling, a parameter region of stable synchronous firing is expanded for lower or higher frequency, respectively.

1. Introduction

In recent neuroscience, many researchers have claimed that synchronization of coupled neurons plays a very important role to code information in the brain. Neurons are classified into two types, class I and class II, by the difference of the onsets of the firing. These two types are confirmed by the physiologic experiments and it is very important to know how these differences affect synchronization.

For mutual synchronization of two neurons, it is clarified by using the phase resetting curve (PRC) that class II neurons easily achieve synchronization [1]-[3]. Also for a large number of neurons with random connections, class II neurons present a good level of synchronization regardless of the connection topology [4]. Also for forced synchronization, class II neurons have advantage of acquiring synchronization [5]. However, Tsuji et al. showed that class I neurons have wider parameter regions of synchronous firing than those for class II neurons by detailed bifurcation analysis [6]. Recently, complex network structure, such as small-world and scale-free is found in real neuronal networks [7]-[10]. It is well known that hubs play a leading role in characterizing the network dynamical behavior [11]-[13]. However, even the dynamics of a single hub is not well studied, especially the properties of peripheral neurons are not identical.

In this paper, we compare parameter regions of synchronization in synaptically coupled Morris-Lecar(ML) neurons with hub structure, when the values of the external current (I_{ext}^1) of the hub and the synaptic conductance are changed. The ML neuron model can be switched between class I and class II excitabilities by changing the value of one parameter [14]. Thus, in this paper, we use the ML model and compare the bifurcation structure for class I and

class II by using the method [15] for analyzing the system with a synaptic delay.

2. Coupled ML Equation

The ML neuron model [16], proposed as a model for describing a variety of oscillatory voltage patterns of Barnacle muscle fibers, is described by

$$C \frac{dV}{dt} = g_{Ca} M_{\infty} (E_{Ca} - V) + g_K N (E_K - V) + g_L (E_L - V) + I_{ext} \quad (1)$$

$$\frac{dN}{dt} = \frac{N_{\infty} - N}{\tau_N} \quad (2)$$

where V is the membrane potential, $N \in [0, 1]$ is the activation variable for K^+ , I_{ext} is the external current and t denotes the time measured in milliseconds. The system parameters E_{Ca} , E_K and E_L represent equilibrium potential for Ca^{2+} , K^+ and leak currents, respectively, g_{Ca} , g_K and g_L denote the maximal conductances of corresponding ionic currents. The V -dependent functions, M_{∞} , N_{∞} and τ_N are given by

$$\begin{aligned} M_{\infty} &= 0.5[1 + \tanh(V - V_a)/V_b] \\ N_{\infty} &= 0.5[1 + \tanh(V - V_c)/V_d] \\ \tau_N &= 1.0/[\phi \cosh(V - V_c/2V_d)] \end{aligned} \quad (3)$$

where V_a and V_c are the midpoint potential at which the calcium current and the potassium current is halfactivated, V_b is a constant corresponding to the steepness of voltage dependence of activation, V_d denotes the slope factor of potassium activation and ϕ is the temperature-like time scale factor.

In this paper, we consider a system of chemically coupled ML neurons. The system equation is described by

$$C \frac{dV_i}{dt} = g_{Ca} M_{\infty i} (E_{Ca} - V_i) + g_K N (E_K - V_i) + g_L (E_L - V_i) + I_{ext}^1 + I_{syni} \quad (4)$$

$$\frac{dN_i}{dt} = \frac{N_{\infty i} - N_i}{\tau_N} \quad (5)$$

$$I_{syni} (\text{hub}) = \sum_{k=2}^n g_{syn} a_k (V_{syn} - V_i) \quad (6)$$

$$I_{syni} (\text{others}) = g_{syn} a_1 (V_{syn} - V_i) \quad (7)$$

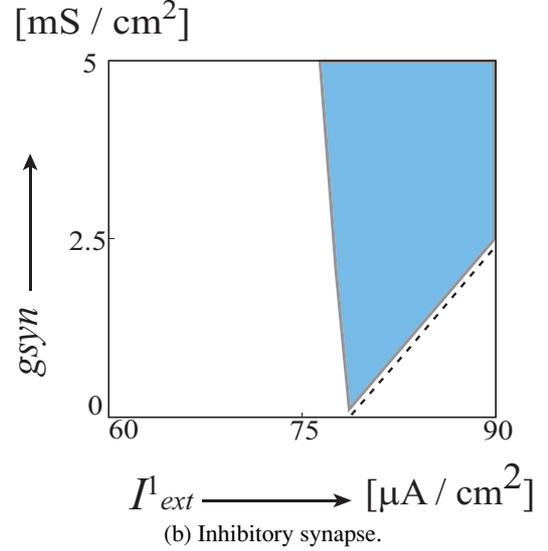
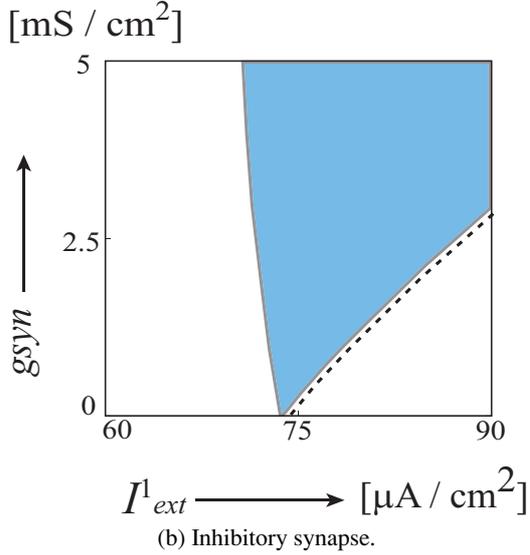
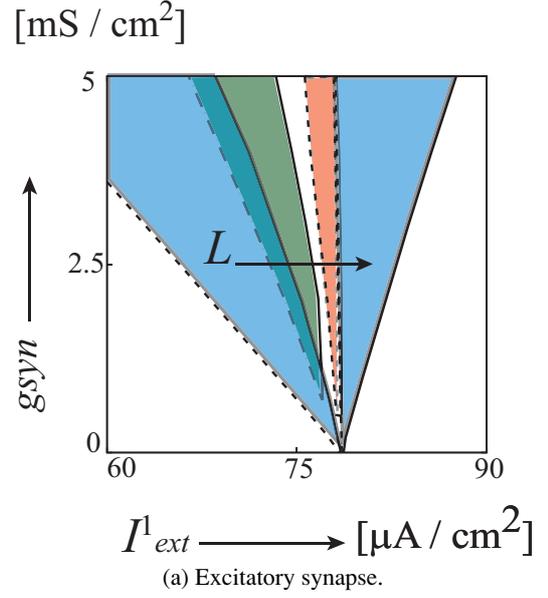
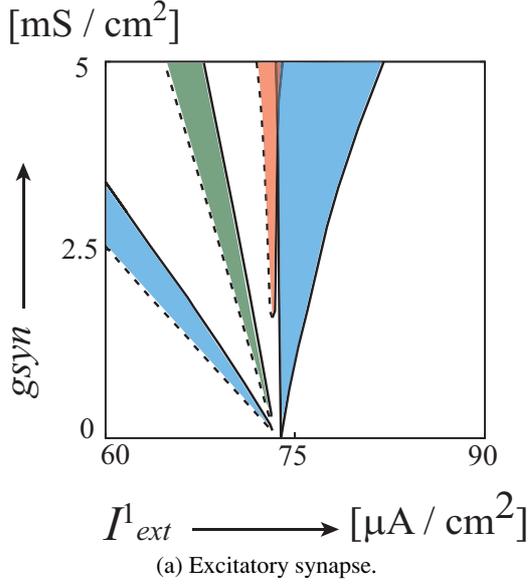


Figure 1: Bifurcation diagrams for class I.

Figure 2: Bifurcation diagrams for class II.

$$\frac{da_i}{dt} = \frac{b_i}{\tau_2} \quad (8)$$

$$\frac{db_i}{dt} = \frac{a_i}{\tau_1} - \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \quad (i = 1, \dots, 6). \quad (9)$$

Note that the solution a_i in Eqs.(8) and (9) with initial condition $(a_i, b_i) = (0, 1)$ at $t = 0$ represents $a_i(t) = \frac{\tau_1}{\tau_2 - \tau_1} \left(\exp(-\frac{t}{\tau_1}) + \exp(-\frac{t}{\tau_2}) \right)$, which is a model for describing the time-dependent conductance of the synapse. Each vector (a_i, b_i) jumps to the constant $(0, 1)$ at $t = t_0^i + \tau_d$ where t_0^i is the time when V_i changes to $V_i > 0$. Namely, the firing information of the neuron transforms to connected neurons with the time delay τ_d , which is mainly caused by the length of synapses.

We set the reversal potential V_{syn} as -60 for inhibitory synapse, or 0 for the excitatory synapse. The aim of this study is to compare parameter regions of in-phase synchronization between class I and class II. The ML model can be controlled between two classes by the value of the parameter V_c . In this paper, we choose $V_c = 12$ and $V_c = 2$ for the class I and the class II neurons, respectively. It is found that the raise time and the decay time constants take different values [17]. In this study, we set the raise time τ_1 as 0.5 (msec), decay time $\tau_2 = 2$ (msec) and $\tau_2 = 7$ (msec) for the excitatory synapse and the inhibitory synapse, respectively. The other parameter values are fixed as shown in Tab.1.

Table 1: The values of parameter in Eq. (1)-(3).

| |
|-------------------------|
| $E_{Ca} = 120[mV]$ |
| $E_K = -80[mV]$ |
| $E_L = -60[mV]$ |
| $g_{Ca} = 4.0[mS/cm^2]$ |
| $g_K = 8.0[mS/cm^2]$ |
| $g_L = 2.0[mS/cm^2]$ |
| $V_a = -1.2[mV]$ |
| $V_b = 18.0[mV]$ |
| $V_d = 17.4[mV]$ |
| $\phi = 1/15[sec^{-1}]$ |
| $C = 20.0[\mu F/cm^2]$ |
| $\tau_d = 1[msec]$ |

3. Results and Discussions

In this study, we fix the values of the external currents $I_{ext}^1, \dots, I_{ext}^6$ as 73.67 or 78.55 $[\mu A/cm^2]$ for the class I or class II neurons, respectively. We show regions of synchronous firing in the parameter plane (I_{ext}^1, g_{syn}) in Figs.1 and 2 obtained by using Yoshinaga's method [15]. In bifurcation diagrams, black solid and black dashed curves indicate the pitchfork and the saddle-node bifurcation, respectively. In blue regions, we observe stable in-phase synchronous firing of five neurons except the hub. Also in green regions, we observe cluster synchronization : in-phase two and three neurons such as (Figs.4(c)). We call this state 3:2. We observe 4:1 synchronized state in orange regions such as(Figs.4(d)).

For the excitatory synapse (Figs. 1(a) and 2(a)), the parameter region of stable synchronous firing is expanded for not only larger but also smaller than $I_{ext}^2 \dots I_{ext}^6$. The appearance of bifurcations strongly depends on both values of the external current and the synaptic conductance. There are some crossover regions for the excitatory synapse in.

On the other hand, for the inhibitory synapse (Figs.1(b) and 2(b)), the bifurcation structure is simple. we only observe in-phase synchronous firing of five peripheral neurons, and these figures show that all peripheral neurons are synchronized when I_{ext}^1 is almost larger than $I_{ext}^2 \dots I_{ext}^6$.

It is known that class II neurons easily achieve synchronization [1]-[3], however our results are opposite to previous those studies. We claim that in some parameter region the class I neurons have advantage of acquiring synchronization.

Figure 3 shows a one-parameter bifurcation diagram along the line L in Fig.2(a). Each number is of in-phase synchronized neurons. From this figure we can see that in Fig.2(a) there are other bifurcation sets and regions of stable states surrounded by their bifurcation sets are at most tri-overlapped. We observe waveforms of membrane potentials shown in Figs.4(a), 4(b), 4(c) and 4(d), on yellow, green, blue and pink line shown in Fig.3, respectively.

4. Conclusion

We investigated synchronization in a system of coupled Morris-Lecar (ML) neurons with hub structure. Homogeneous (coupled the same class neurons) case for the class I and the class II neurons is considered for both the excitatory and the inhibitory synapses. We compared synchronization phenomena between two classes, and found that for the excitatory or inhibitory coupling, the parameter region of stable synchronous firing is expanded for lower or higher frequency, respectively. In the future works, we should clarify the reason why the class I neurons synchronize more easily than the class II neurons. Also heterogeneous (coupled the different class neurons) case should be considered for both excitatory and inhibitory synapses.

References

- [1] B. Ermentrout, M. Pascal and B. Gutkin, The effects of spike frequency adaptation and negative feedback on the synchronization of neural oscillators, *Neuro Comput.*, 13: 1285-1310, 2001.
- [2] B. Ermentrout, Type I membranes, phase resetting curves, and synchrony, *Neuro Comput.*, 8: 979-1001, 1996.
- [3] J. Rinzel and B. Ermentrout, Analysis of Neural Excitability and Oscillations, in *Methods in Neuronal Modeling*, eds.C.Koch, and I.Segev, The MIT Press, 1998.
- [4] L.F. Lago-Fernandez, F.J. Corbacho and R.Huerta, Connection topology dependence of synchronization of neural assemblies on class 1 and 2 excitability, *Neural Networks*, 14: 687-696, 2001.
- [5] E. Ioka and H. Kitajima, Forced synchronization of class I and class II neurons, *Journal of Signal Processing*, vol.11, no.11, pp.111-115, Jan. 2007.
- [6] S. Tsuji, T. Ueta, H. Kawakami and K. Aihara, Bifurcation in modified BVP neurons coupled by gap-junctions, *Proc. NCS'04*, 495-498, Hawaii, USA, 2004.
- [7] O. Sporns, D. R. Chialvo, M. Kaiser, and C. C. Hilgetag, Organization, development and function of complex brain networks, *TRNENDS in Cognitive Science* 8, 419 (2004).
- [8] D. S. Bassett, A. M. Lindenberg, S. Achard, T. Duke and E. Bullmore, Adaptive reconfiguration of fractal small-world human brain functional networks, *Proc. Natl. Acad. Sci. U.S.A.* 103, 19518 (2006).
- [9] L. Zemanová, C. Zhou and J. Kurths, Structural and functional clusters of complex brain networks, *Physica D*, 224:202-212, 2006.
- [10] K.V. Srinivas, R. Jain, S. Saurav and S.K. Sikdar, Small-world network topology of hippocampal neuronal network is lost, in an in vitro glutamate injury model of epilepsy, *Eur. J. Neurosci.* 25: 3276-3286, 2007.
- [11] D. Huang and G. Pipa, Achieving synchronization of networks by an auxiliary hub, *EPL* 77, 50010 (2007).

- [12] N. Masuda and N. Konno, VIP-club phenomenon: Emergence of elites and masterminds in social networks ,Social Networks 28, 297 (2006).
- [13] P. N. McGraw and M. Menzinger, Topology and computational performance of attractor neural networks ,Phys. Rev. E 68, 047102 (2003).
- [14] K. Tsumoto, H. Kitajima, T. Yoshinaga, K. Aihara and H. Kawakami, Bifurcations in Morris-Lecar neuron model, Neurocomputing, 69(4-6): 293-316,2006.
- [15] T. Yoshinaga, Y. Sano and H. Kawakami, A method to calculate bifurcations in synaptically coupled Hodgkin-Huxley equations, International Journal of Bifurcation and Chaos, 9(7): 1451-1458,1999.
- [16] C. Morris and H. Lecar, Voltage oscillations in the barnacle giant muscle fiber, Biophys. J. 35 (1981) 193-213.
- [17] T. Tateno and H.P.C. Robinson, Rate Coding and Spike-Time Variability in Cortical Neurons With Two Types of Threshold Dynamics, J Neurophysiol 95: 2650-2663, 2006.
- [18] E. Ioka, H. Kitajima and K. Tsumoto, Synchronization of Bidirectionally Coupled Neurons with Asymmetrical Inputs, Vol.106, No.452, pp. 69-72 NLP2006-138.

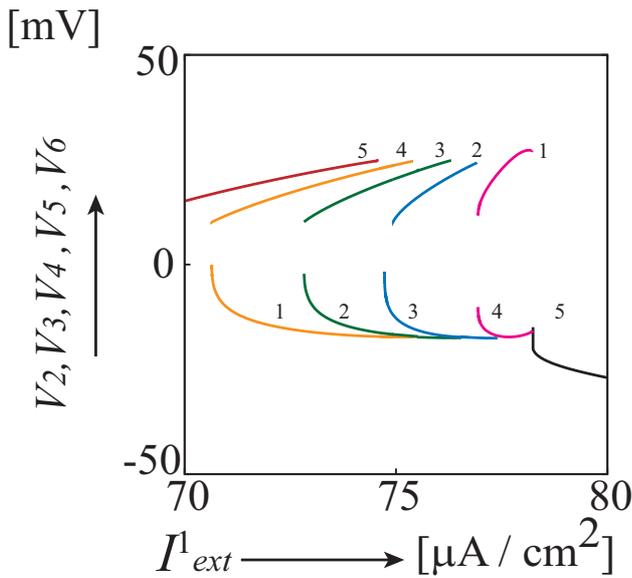
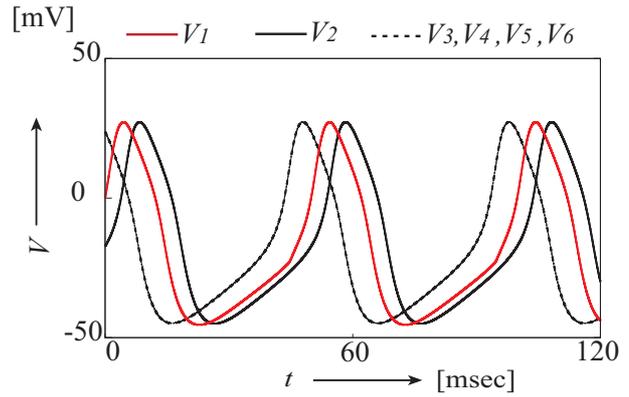
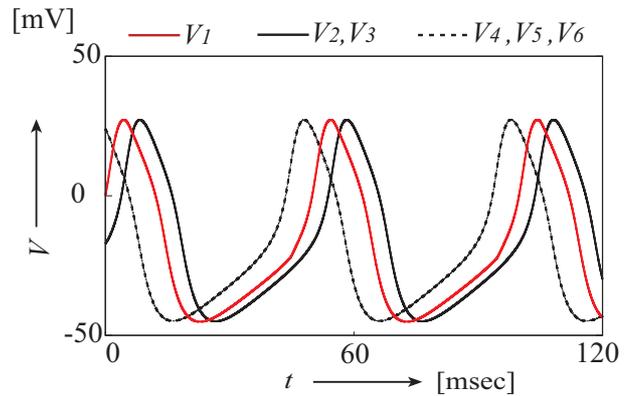


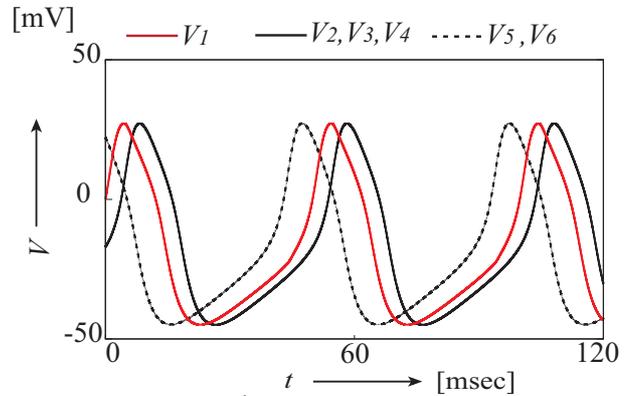
Figure 3: One-parameter bifurcation diagram. Complete synchronization : red and black, 4:1 cluster synchronization : yellow and pink, 3:2 cluster synchronization : green and blue.



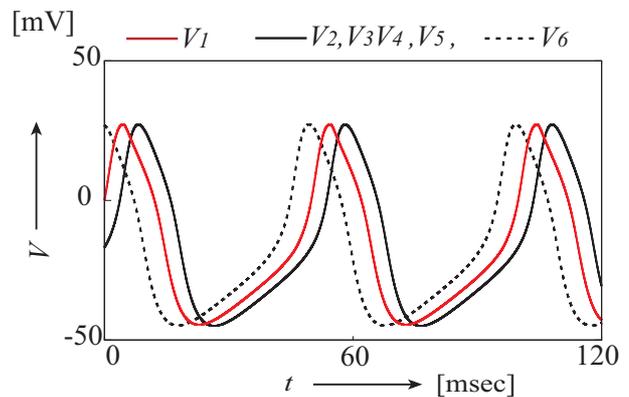
(a) $(I_{ext}^1, g_{syn}) = (75, 2.5)$.



(b) $(I_{ext}^1, g_{syn}) = (76, 2.5)$.



(c) $(I_{ext}^1, g_{syn}) = (76.5, 2.5)$.



(d) $(I_{ext}^1, g_{syn}) = (78, 2.5)$.