Exponential Transient Rotating Waves and Their Bifurcations in a Ring of Unidirectionally Coupled Bistable Lorenz Systems

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Ring of unidirectionally coupled bistable Lorenz systems

\[
\begin{align*}
\frac{dx_n}{dt} &= \sigma(y_n - x_n) \quad (\sigma = 10) \\
\frac{dy_n}{dt} &= \rho x_{n-1} - y_n - x_n z_n \quad (1 \leq \rho \leq 12) \\
\frac{dz_n}{dt} &= -\beta z_n + x_n y_n \quad (\beta = \frac{8}{3})
\end{align*}
\]

\((1 \leq n \leq N, \quad x_0 = x_N)\)

Various rotating waves
  · Exponential transient rotating waves
  · Chaotic rotating waves
Bifurcation diagrams when $N = 9$

### (a) steady states

Spatially uniform steady states:

$$x_n = y_n = \pm [\beta (\rho - 1)]^{1/2}, \quad z_n = \rho - 1 \quad (1 \leq n \leq N)$$

Stable states of a single Lorenz system

→ Destabilized through the Hopf bifurcation

The origin $(x_n = y_n = z_n = 0 \quad (1 \leq n \leq N))$

### (b) rotating wave 1

Various rotating waves are generated from the origin.

### (c) rotating wave 2

Exponential transient rotating waves

Chaotic rotating waves
Mean duration $m(T)$ of transient rotating waves increases exponentially with the number $N$ of elements.

$$m(T) \propto \exp(N)$$  (Exponential transients)

Unidirectional coupling causes chaotic rotating waves of intermittent type.

\[
(x_1(t_k) = 0, \frac{dx_1(t_k)}{dt} > 0)
\]